▪ Journal of Competitiveness

Hybrid wavelet adaptive neuro-fuzzy tool supporting competitiveness and efficiency of predicting the stock markets of the Visegrad Four countries

Zuzana Janková

Abstract

The stock market is influenced by many factors and predicting its development is a difficult and complicated task. Successful and accurate stock market prediction is absolutely essential for countries' economies as it helps to create a competitive advantage for listed companies through economies of scale. The aim of the paper is to decompose and denoise stock time series using wavelet analysis, detect a smoothed trend and predict future development using an adaptive neuro-fuzzy model. This hybrid fusion model is also referred to as the WANFIS model. The application of the WANFIS model is carried out on less developed stock markets, specifically on the official stock market indices of the Visegrad countries, namely the Czech Republic, Slovak Republic, Poland and Hungary. Recently, wavelet analysis has been among the most promising mathematical tools, which can be used to easily decompose continuous signals or time series in the time and frequency domains. The results show that the proposed WANFIS hybrid model demonstrates a more accurate prediction of the development of stock indices than individual models alone. Experimental results show that the fusion model provides a promising and effective tool for predicting even less liquid and less efficient stock markets, such as those in the V4 countries. A useful and accurate prediction alternative proven in emerging stock markets is offered.

Keywords: *ANFIS, neuro-fuzzy model, stock market, V4 countries, Visegrad countries, wavelet analysis, wavelet transformation*

JEL Classification: C53, C63, G11

Received: December 2022; 1st Revision: January 2023; Accepted: February 2023

1 INTRODUCTION

Due to its importance in economies, the prediction of stock market development has become an attractive topic in recent years. There is a need to cover the urgent need to predict the future behavior of the stock market with the aim of eliminating negative consequences of investment risks, and thereby increasing the competitive advantage of companies. A competitive advantage, as stated by Forsyth & Mongrut (2022), can be considered the ability of companies to generate permanent above-standard returns that are higher than the cost of capital. If a company is distinguishable from competing companies, it gains a competitive advantage, thanks to which it is able to generate additional revenue. However, predicting the development of the stock market is a very complex process, as these markets are characterized by chaotic behavior and unstable development. A useful and effective tool that is able to accurately and timely predict the development of the stock market can help companies gain a competitive advantage, as it allows timely and accurate detection of sharp changes in market development. This enables companies to respond to changing conditions earlier than their competitors.

For this reason, it is necessary to use more sophisticated tools for the analysis of stock markets, which can cover these issues. To estimate the future behavior of stock markets, it is appropriate to use artificial intelligence methods, which are currently on the rise not only in this area. To

https://doi.org/10.7441/joc.2023.01.04

solve the second issue regarding the type of investor, it is necessary to use wavelet analysis, which integrates both time and frequency domains. This means that if stock market fluctuations and performance vary depending on frequency, the risk will also be different for each type of investor. Although ripples are more popular in areas such as signal and image processing, meteorology, and physics, they can also provide fruitful knowledge from an economic or financial perspective. Ramsey (1999) describes that economic and financial phenomena can have different characteristics at different time scales, and therefore wavelet analysis tools make it possible to explore the multilevel features of these phenomena. Unlike Fourier and spectral analysis, wavelet analysis is localized in time and scale. The motivation to use the wavelet approach is mainly the fact that it provides an effective and convenient way of representing complex variables. In addition, this approach is useful for detecting seasonal and cyclical fluctuations in financial time series and structural breaks, as well as for analyzing trends. One of the main advantages of continuous wavelet analysis, as reported by Rua & Nunes (2009) and Lin et al. (2018), is that it is not necessary to define the number of waves (time scales), as this number is derived according to the length of the time series. This makes it easy to interpret hidden patterns or detected information. For that reason, continuous wavelet analysis is applied to evaluate mutual links and relationships in the time and frequency domain.

We propose a new hybrid wavelet adaptive neuro-fuzzy tool (WANFIS) supporting competitiveness and efficiency in predicting the stock markets that could produce a more accurate and concise stock market forecast based on wavelet transform analysis and a neuro-fuzzy approach. This innovative method provides promising results based on previous empirical studies (Kumar Chandar, 2019; Alenezy et al., 2021). In the above papers, the prediction is made for individual stock market titles. The novelty of the presented research is the focus on the less liquid and less efficient stock markets of Central European countries. So, the focus is on small European economies. Furthermore, the prediction is made for stock indices and not for selected stock titles, which are de facto proxies of the entire stock market of the individual countries, and thus provide a higher indicative value with regard to the development of other financial markets, i.e., economies as a whole. To the best of the authors' knowledge, the WANFIS model has not yet been adequately applied to these markets. The main aim of the paper is to predict the development of the main stock indices of Central European countries, namely the Czech Republic, Slovakia, Poland and Hungary – also generally referred to as the "Visegrad Group," "Visegrad Four" or "V4" – by means of a hybrid wavelet adaptive neuro-fuzzy model (WANFIS). To fulfill this goal, it is necessary to first (1) detect the intercorrelation of selected Central European stock markets through wavelet coherence, both in the time and frequency domains; (2) apply a continuous wavelet transform to denoise and decompose the overall trend of stock indices; (3) use the smoothed trend as an input to an artificial intelligence model, specifically to an expert system based on fuzzy logic integrating neural networks, which is also referred to as an adaptive neuro-fuzzy inference system (ANFIS). Using fuzzy logic, fuzzy membership functions are set up and the artificial neural network defines the knowledge base of rules based on which the prediction of selected V4 stock indices is determined.

The paper is arranged as follows: In Section 2, a critical literature search is performed, summarizing the findings in the subject area. Section 3 contains the methodology of wavelet analysis, which is used to smooth the analyzed time series, and an adaptive neuro-fuzzy system is used to predict stock indices. In Section 4, the WANFIS model is applied, and the empirical findings are presented, including a discussion. In Section 5, there is a conclusion synthesizing the obtained results and possible future directions of subsequent research.

https://doi.org/10.7441/joc.2023.01.04

2 THEORETICAL BACKGROUND

Prediction of the development of the price and volatility of stock markets is undoubtedly a very interesting topic important for business decision-making. As Kristjanpoller & Michell (2018) add, this is becoming a hot topic, especially in developing economies. In these countries, according to Pabuçcu & Değirmenci (2018), investors are exposed to high risk and uncertainty regarding the future development of the markets. For that reason, the investment strategy, or the tool integrating this uncertainty, plays an important role in the competitive environment. It seems that an appropriately chosen investment strategy can become a major competitive advantage for many companies (Xu et al., 2022). In financial markets, people and companies can apply different kinds of models and tools to generate investment decisions. Developing economies or developing stock markets show certain specificities that make it necessary to use an appropriate and effective approach that is able to grasp these unique features.

In particular, fuzzy logic is suitable for modeling the inclusion of uncertainty and ambiguity in the stock market. The application of the neuro-fuzzy model can be found in the research of Janková & Rakovská (2022), who proposed a model that facilitates investment decisions in financial assets. Rajab & Shama (2019) compared the model with an adaptive neuro-fuzzy system, artificial neural networks, and classical statistical techniques, such as multiple regression analysis and GARCH. Based on the obtained results, it can be stated that the proposed ANFIS model is a suitable tool especially for modeling complex and nonlinear problems. In recent years, the popularity of the application of wavelet analysis in the field of finance has specifically increased the interest in the use of wavelet decomposition and approximation to smooth stock time series. As described by Dima et al. (2015), wavelet analysis can provide a flexible tool for analyzing less stable or unstable time series such as the financial markets. The application of wavelet analysis can be found, for example, in the study by Janková (2020), who proposes to use wavelet analysis to determine the relationship between the stock index and investor sentiment. The strong relationship and correlation between stock markets identified by wavelet coherence slowly decrease over time. This output indicates that the relationship is undervalued or overvalued in a short period of time. In addition, the author found that sentiment and stock markets show a stronger correlation during a crisis. This was illustrated, for example, during the COVID-19 pandemic. Similarly, Dash & Maitra (2018) examine the relationship between investor sentiment and the Indian stock market. The wavelet method was used to decompose the variables of sentiment and return of stocks into different time-frequency domains. The study provides support for the fact that, whether investors are short-term or long-term, their investment activities cannot be deprived of sentiment. The purpose of the paper by Tiwari et al. (2016) is to assess the level of joint movement between the PIIGS stock markets and the markets of the United Kingdom and Germany. The authors applied a continuous wavelet transformation for analysis in the time-frequency domain of the movement of market returns of stock indices. The research output found that during the financial crisis, the correlation is high in the short term. In the long-term time horizon, mutual movements are present across the entire monitored period.

In addition to the above, there is a fusion of wavelet analysis and artificial intelligence methods, especially neural networks and fuzzy logic. The integration of these methods leads to more accurate outputs than the individual models alone. Evidence can be found, for example, in Zarei et al. (2018), whose study compared the power of a neuro-fuzzy model with a wavelet neuro-fuzzy model in the prediction of prices of bank stock on the Tehran Stock Exchange. The research results showed that the model with the implementation of wavelet analysis provides higher performance compared to the model without wavelet analysis. The accuracy of a wavelet neuro-fuzzy model is 90% and above; the accuracy of a neuro-fuzzy model is over 80%. It can be stated that a wavelet neuro-fuzzy model is more reliable than a neuro-fuzzy model. Similarly,

https://doi.org/10.7441/joc.2023.01.04

Artha et al. (2018) combined wavelet transform and neuro-fuzzy techniques used to predict daily closing stock price data. The use of the wavelet transformation and its integration into the neuro-fuzzy model can, according to the authors' output, provide a highly accurate prediction of the price of stocks. However, the authors add that before performing a wavelet analysis, it is necessary to consider which family of wavelet filters to use and what type of wavelet transform to use. Raoofi & Mohammadi (2018) apply an adaptive neural fuzzy inference system reduced by the wave decomposition of random noise; therefore, it reduces errors and improves the required prediction of a chaotic time series. Their results indicate the superiority of the proposed method compared to others. Kumar Chandar (2019) formulated a prediction model of the time series using mergers of the network-based wavelet adaptive fuzzy inference system (WANFIS), which is able to predict the stocks. According to experimental results, the proposed fusion model shows a higher accuracy of development prediction than separate models. According to the author, WANFIS thus offers an interesting alternative that can be used on stock markets with promising prediction results and can thus become a useful tool in the economic field.

Based on the impressive results of the fusion of the wavelet analysis and the neuro-fuzzy model in recently published studies, the present work deals with the WANFIS model or wavelet adaptive neuro-fuzzy inference system for stock markets. Specifically, the authors of the paper are not aware of a similar application to the stock markets of Central European countries. The Visegrad 4 area has been chosen because it is an atypical stock market, different from international markets, which show high liquidity and efficiency. According to a study by Aloui et al. (2018), their outputs identify differences in patterns for emerging and developed markets, making the empirical results very relevant to experts and policymakers. For this reason, this paper is based on a survey of the outcome of the hybrid model in emerging markets. The aim of the paper is to examine whether the hybrid model will also achieve excellent results in these less liquid and less efficient markets in the Czech Republic, Poland, the Slovak Republic, and Hungary.

3 RESEARCH OBJECTIVE, METHODOLOGY AND DATA

3.1 Wavelet Decomposition

Wavelet analysis is a promising mathematical tool that is used to divide time series in continuous time into components of various scales. Wavelet Transform (WT) provides time series decomposition not only in the time domain but also in the frequency domain, even if the data are nonstationary. The benefits of WT can be seen in several respects. Compared to Fourier transforms, WT is preferable to apply if the signal or time series shows a non-periodic character with sharp peaks and occurrences of discontinuity. Furthermore, the ripples define the final domain, which is well localized with respect to time and frequency. This characteristic allows its good use in the study of nonstationary signals. Finally, wavelet analysis significantly shortens the processing time and speeds up the calculation, according to Kumar Chandar (2019). According to Li and Tam (2017), the basic idea of the wavelet denoising model can be defined as:

$$f(t) = x(t) + \varepsilon(t) \tag{1}$$

where $f(t)$ is an observed signal, $x(t)$ is a real signal and $\varepsilon(t)$ is the white noise. Wavelet denoising is based on filtering out the largest possible part of $\varepsilon(t)$.

In general, WT can be divided into two basic types: continuous wave transformation (CWT) and discrete wave transformation (DWT); as stated by Lai & Liu (2014), the calculation of wavelet coefficients in CWT is a computationally lengthier process. Xu et al. (2017) describe the wavelet transform and its decomposition process. The CWT can be generated by integrating the mother wavelet $\psi(t)$ into the time series $x(t)$. Thanks to the translation and dilation of the mother wavelet, a two-dimensional time series is created. The mother wavelet can be written in a mathematical formula as follows:

$$\psi_{t,s}(t) = \frac{1}{\sqrt{s}}\psi(\frac{t-\tau}{s}) \tag{2}$$

where τ indicates the time location of the mother wavelet, and s is a scale parameter. A scale parameter greater than one means that the wavelet records rapid changes at high frequencies. While a scale parameter smaller than one captures slow changes at low frequencies. With respect to the defined parameters, the CWT is written as follows:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t)\psi_{t,s} * (t)dt \tag{3}$$

where $\psi_{t,s} * (t)$ represents a complex conjugate basic wavelet $\psi_{t,s}(t)$. The most commonly used mother wavelet for the analyzing phase is the Morlet wavelet (Rua, 2012).

$$\psi_{\omega_0}(t) = \pi^{-\frac{1}{4}}(e^{\omega_0 t} - e^{\frac{-\omega_0^2}{2}})e^{-t^2/2} \tag{4}$$

In the above equation $\pi^{-\frac{1}{4}}$ z ensures the conditions of admissibility of the mother wavelet. If $\omega_0 > 5$, the Morlet wavelet is defined as follows:

$$\psi(t) = \pi^{-\frac{1}{4}}e^{\omega_0 t}e^{-t^2/2} \tag{5}$$

where the parameter ω_0 in eq. (4) and (5) indicates the number of oscillations within the Gaussian envelope. With this parameter, one chooses between two conflicting goals. One is frequency localization, and the other is time localization. A smaller value of the parameter indicates better localization in time and worse localization of frequencies. A parameter setting of six is usually chosen, which will guarantee a balance between the two goals.

$$\sigma_x^2 = \frac{1}{C_\psi}\int_0^{+\infty}\int_{-\infty}^{+\infty}|W_x(\tau, s)|^2\frac{d\tau ds}{s^2}, with\ 0 < C_\psi = \int_0^{+\infty}\frac{|\hat{\psi}(\omega)|^2}{\omega}d\omega < \infty \tag{6}$$

where $\hat{\psi}(\omega)$ is the Fourier transform. The cross-wavelet transform of two time series $x(t)$ and $y(t)$ can be written as $W_{xy}(t, s) = W_x(t, s) * W_y(t, s)$.

3.2. Adaptive Neuro-Fuzzy Inference System

The Adaptive Neuro-Fuzzy Inference System (ANFIS) is a hybrid model that combines fuzzy logic (FL) and artificial neural networks (ANN). Thanks to this combination of the two methods, ANFIS can take advantage of both of them. Especially due to fuzzy logic, the model is better understood and more similar to human thinking, as it contains linguistic expressions rather than numerical calculations and contains a knowledge base of rules that is easy to understand even for laymen, as described by Janková et al. (2021). By integrating neural networks, the ANFIS model is more transparent to users and causes fewer memory errors; in addition, it is able to adaptively work with nonlinear problems and can easily learn from the provided data sources. The basis of the ANFIS model is the construction of a fuzzy system of the Sugeno type, whose membership function parameters are derived from training examples and is computationally more efficient than another type of Mamdani used, which is more dependent on expertise, according to Şahin & Erol (2017).

The principle of operation of the ANFIS model is described in Mathur et al. (2016). In short, the input data is converted via fuzzification to fuzzy membership functions, which are connected to a neural network block. In this block of neural networks, a knowledge base is constructed through a set of rules that is connected to the inference module. To train this model, a back-propagation algorithm is chosen, based on which the right rules are selected to ensure the optimal output. The architecture of the ANFIS model contains five layers. Individual layers have nodes that use transfer functions to transfer fuzzy inputs. Finally, the fuzzy output generated from the model is converted to a sharp output through the defuzzification block. Such a framework makes modeling with ANFIS more systematic and less dependent on the expertise of the expert group or experts. Two fuzzy IF-THEN rules based on the Sugeno first-order model are considered to introduce ANFIS:

$$\text{If } (x \text{ is } A_1) \text{ and } (y \text{ is } B_1) \text{ then } (f_1 = p_1 x + q_1 y + r_1) \tag{7}$$

$$\text{If } (x \text{ is } A_2) \text{ and } (y \text{ is } B_2) \text{ then } (f_2 = p_2 x + q_2 y + r_2) \tag{8}$$

where x and y are inputs, A_i and B_i are fuzzy sets, f_i are outputs in the fuzzy area specified by fuzzy rules, and consequent parameters p_i, q_i and r_i are determined during the training process.

It can be seen that in this ANFIS architecture there are two adaptive layers, the first layer and the fourth layer. In the first layer, there are three modifiable parameters $\{a_i, b_i, c_i\}$ called premise parameters that relate to the input membership functions. In the next layer, the parameters $\{p_i, q_i, r_i\}$ are referred to as consequent parameters to the first-order polynomial. In order for the output of the ANFIS model to correspond with the training data set, the goal of the learning algorithm is to tune the premise and consequent parameters. When the premise parameters $\{a_i, b_i, c_i\}$ of the membership function are fixed, the ANFIS output can be written as follows:

$$f = \frac{w_1}{w_1+w_2} f_1 + \frac{w_2}{w_1+w_2} f_2 \tag{9}$$

If the premise parameters are not fixed, the training of the model will slow down as the space of convergence will increase. To solve this problem, the gradient descent method and the smallest squares method can be combined, while either the forward or backward pass method can be used. The first method is used to optimize consequent parameters with premise parameters fixed. The backward pass is used to optimally adjust the premise parameters corresponding to the fuzzy sets in the input domain. The ANFIS output is calculated using the following parameters found in the forward pass. The output error is used to match the premise parameters using the standard back propagation algorithm. This hybrid algorithm has been shown to be highly effective in training ANFIS, according to Jang (1992) and Jang (1993).

4 EMPIRICAL RESULTS AND DISCUSSION

4.1. Data Description

For the application of the WANFIS model, data from the stock markets of four European countries are selected, which are referred to as the so-called Visegrad countries. Namely, these are stock markets in the Czech Republic, Poland, Hungary and Slovakia, and these markets are represented by leading stock indices listed on local stock exchanges. The official index of the Prague Stock Exchange is the PX index, and the Bratislava Stock Exchange is SAX. The development of both indices is shown in Figures 1 and 2, respectively. The Warsaw Stock Exchange can be considered the largest stock exchange in the eastern part of Europe. The WIG stock index is listed here, which is a leading stock index. Figure 4 shows the development of the WIG index in the given period. The second largest stock exchange in Central Europe is the Hungarian Budapest Stock Exchange, not only in terms of market capitalization but also in terms of liquidity. The stock index listed on this exchange is the BUX stock index, the development of which is shown in Figure 3.

Fig. 1 – Empirical Dynamics of PX. Source: PSE (2020)

Fig. 2 – Empirical Dynamics of SAX. Source: BSSE (2020)

Fig. 3 –Empirical Dynamics of BUX. Source: BSE (2020)

Fig. 4 –Empirical Dynamics of WIG. Source: GPW (2020)

Basic descriptive statistics of daily returns on selected stock indices of the Visegrad 4 countries are presented in Tab. 1. It can be noted that the average return and median stock indices for the period under review are almost identical and are around zero. The highest maximum return can be seen in the stock index SAX, which reached 9.55%, followed by the Hungarian BUX index, with a maximum yield of 5.09%. By contrast, the lowest maximum return of 3.05% was achieved by the Polish WIG. In terms of minimum return, the Slovak stock index SAX shows the largest loss of -8.91%. The other stock indices reached a minimum return or maximum loss of about -5% on average. A simple measure, which is the standard deviation, is most often used to measure the volatility of stock indices. Based on the above description, it is clear that the Slovak stock index SAX, whose standard deviation is 1.13%, shows the biggest fluctuations. However, other V4 indices show a volatility of around 1% over the period under review. The skewness indicator is used to measure asymmetry or, in other words, lack of symmetry. Based on the values, it can be concluded that stock indices in all countries surveyed have an asymmetric probability distribution of yields. If the kurtosis values fluctuated around a value of 3, this would indicate a Gaussian probability distribution. However, none of the monitored time series reaches this value. This indicates that the probability distribution of the stock indices of the V4 countries does not show a normal distribution. A Jarque-Bera test is performed to test this assumption. As part of this test, a null hypothesis assuming a Gaussian probability distribution (chi-square with 2 degrees of freedom) is defined. The output values of the test reject the null hypothesis for all monitored stock indices at the 1% significance level. As Polanco-Martinez (2018) notes, this is a completely normal phenomenon that is consistent with previous findings on stock market returns.

Tab. 2 shows the correlation matrix obtained using Spearman correlation coefficients between the stock indices of the V4 countries. The maximum correlation occurs for two SAX-BUX indices with a value of 0.8842 and WIG-PX with a correlation coefficient value of 0.7966. On the other hand, the minimum correlation occurs for PX-SAX with 0.3365. The mean dependence rate is then observed with the pairs SAX-WIG, WIG-BUX, and BUX-PX. To some extent, this suggests that the Visegrad countries are interrelated, and the neighboring countries correlate.

Tab. 1 – Summary statistics of selected stock indexes. Source: own research

Stock index	SAX	WIG	BUX	PX
Country	Slovakia (SK)	Poland (PL)	Hungary (HU)	Czech Republic (CZ)
Number of obs.	1240	1240	1240	1240
Mean	0.0005	0.0001	0.0007	0.0000
Median	0.0000	0.0003	0.0008	0.0004
Standard deviation	0.0113	0.0090	0.0106	0.0083
Minimum	-0.0891	-0.0566	-0.0607	-0.0460
Maximum	0.0955	0.0305	0.0509	0.0457
Skewness	0.1889	-0.4827	-0.1384	-0.4015
Kurtois	9.7729	2.7458	2.0330	3.4318
J-B test	4942.0813**	437.6870**	217.4970**	641.8113**

** indicates p-value lower than 0.01

A statistical technique called correlation analysis is applied to investigate dependencies and correlations between market returns of stock indices. According to Pinho & Madaleno (2010) or Rua & Nunes (2009), correlation analysis is used to detect common movement between stock market returns over time in the context of different frequencies or periods. Livan et al. (2012) showed that using the standard Spearman estimate to calculate the correlation coefficients between stock indices in case of nonstationary behavior can be complicated. For that reason, it is necessary to use a statistical apparatus that is able to work with the nonstationary behavior of the time series. One such approach is the wavelet analysis, which involves correlation across the time and frequency domains.

Tab. 2 – Correlation matrix of selected stock indexes. Source: own research

Stock index	SAX	WIG	BUX	PX
Country	Slovakia (SK)	Poland (PL)	Hungary (HU)	Czech Republic (CZ)
SAX	1	0.4077*	0.8842*	0.3365*
WIG	0.4077*	1	0.6085*	0.7966*
BUX	0.8842*	0.6085*	1	0.4635*
PX	0.3365*	0.7966*	0.4635*	1

The * symbol at the 95% significance level highlights statistically significant index correlations.

Prior to the wavelet decomposition itself, a wavelet coherence is performed, showing the dependencies and interconnections of stock indices in the time-frequency domain of Visegrad 4. The visualization is captured in Figure 5 and is divided into individual time series pairs represented by the respective stock indices. Red indicates the highest strength and dark blue is the lowest. The area within the contours indicates significant coherence at the 5% level of significance. The figures show the time period on the horizontal axis, while the frequency components are captured on the vertical axis and are converted to time units, in this case, to months. The arrows inside the graphics display can be used to analyze the positive or negative relationship of stock indices. Within the BUX-SAX, PX-SAX, PX-WIG and SAX-WIG indices, arrows pointing to the right can be indicated in recent months. This means that the given stock indices are in a phase as positively correlated. On the other hand, pointing the arrows to the left would indicate a negative correlation, which is evident, for example, with BUX-SAX in the early phases of the observed time period.

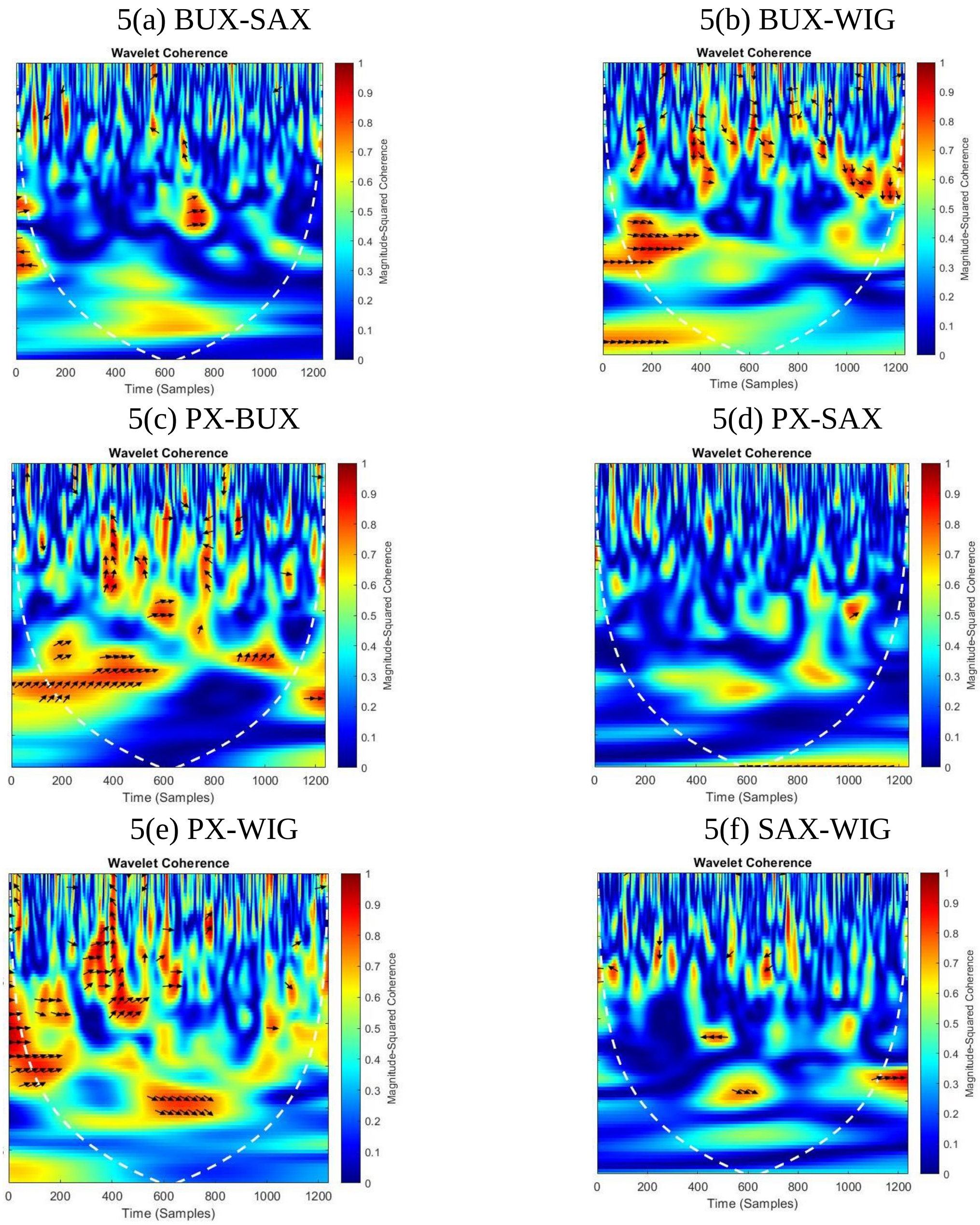

Fig. 5 – Wavelet Coherence. Source: own research.

The short-term horizon, which is represented by higher frequencies, corresponds to the behavior of short-term investors. In contrast, low frequencies correspond to the long-term horizon and are typical of long-term investors. The figure shows the low coherence between stock indices, especially between the BUX, SAX and PX indices. However, a different view offers a comparison of these indices with the TIG index. Especially in the last monitored months, most stock indices show coherence at higher frequencies of 64 and higher. The downward direction of the arrows from BUX, PX, and SAX to the TIG indicates that the TIG index leads the BUX, PX, and SAX index.

4.2. Wavelet Decomposition and Denoising

Wavelet analysis is used for the decomposition and subsequent smoothing of the time series of leading stock indices listed on the Czech, Slovak, Hungarian and Polish stock markets. Wavelet decomposition is used to detect the development trend of selected stock indices. Various discrete waves can be used for this purpose, e.g., the family of Daubechies, Harr, coiflets, and symlets, depending on the characteristics and nature of the time series. In this study, a discrete Daubechia wave is chosen because it provides compact support and is particularly suitable for the analysis of high-grade problems. Figures 6 to 9 show the wavelet decomposition of the analyzed stock indices SAX, WIG, BUX and PX, the decomposition being performed up to level 5. The individual details of the time series represent a variation of the time scale and the prices of the stock indices. Different levels of degradation correspond to time scales: d1 (2–4 days), d2 (4–8 days), d3 (8–16 days), d4 (16–32 days) and d5 (32–64 days).

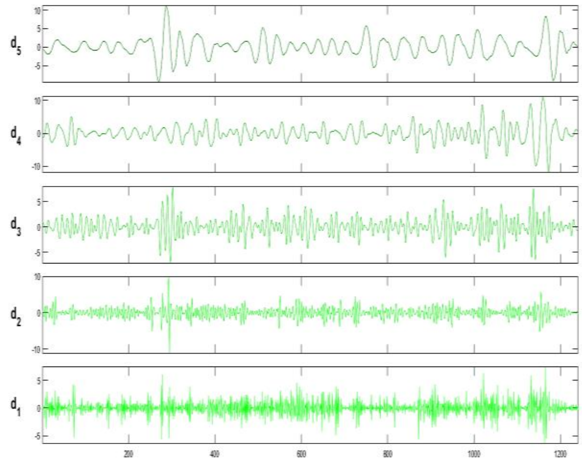

Fig. 6 – Decomposition of SAX index in detail. Source: own research.

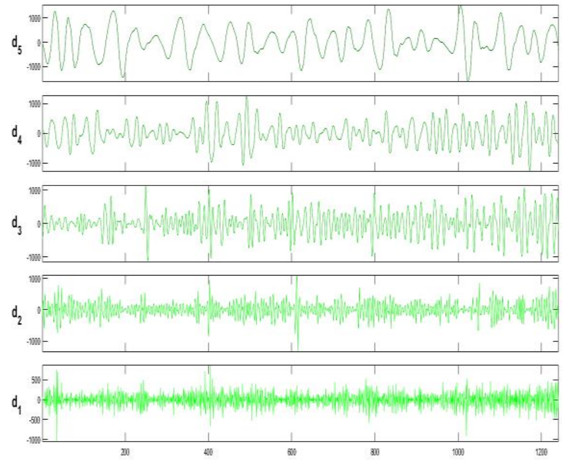

Fig. 7 –Decomposition of WIG index in detail. Source: own research.

The components obtained by the wavelet decomposition d1 and d2 represent a short-term horizon or high fluctuation frequency and represent stock market fluctuations due to shocks that occur in the time scales of 2 to 4 days and 4 to 8 days, i.e., fluctuations within one week. The wavelet component d3 represents the medium-term horizon and depicts shocks caused in the time horizon of 8 to 16 days, i.e., fluctuations within two weeks. The long-term horizon is presented by the wavelet decomposition d4, resp. d5, which represents a time period of 16 to 32 days, i.e., approximately one month, resp. 32 to 64 days, i.e., about two months.

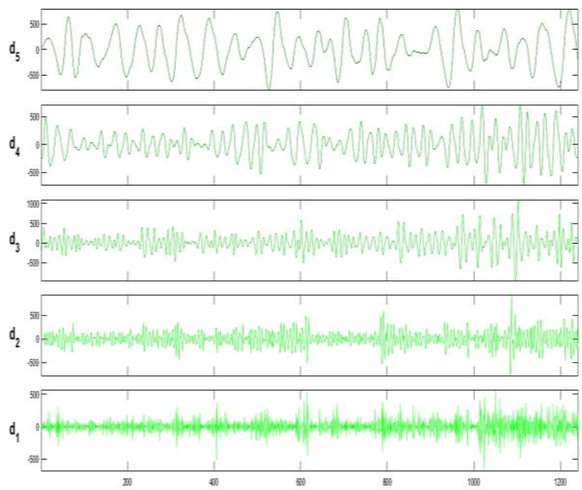

Fig. 8 – Decomposition of BUX index in detail. Source: own research.

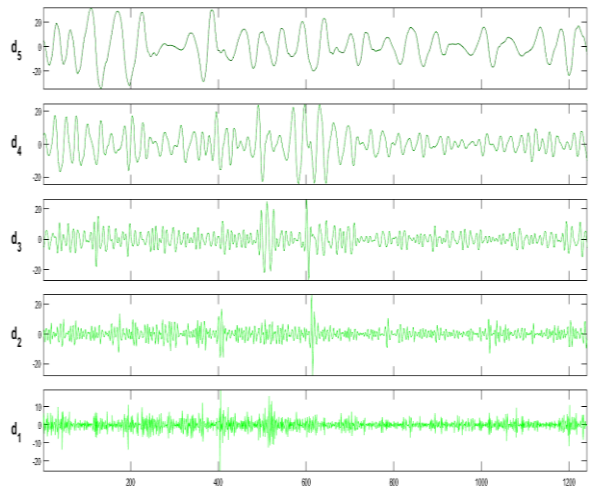

Fig. 9 – Decomposition of PX index in detail. Source: own research.

Wavelets are a good tool for detecting sudden changes in a signal. The waves of the first and second levels (d1 and d2) capture this discontinuity clearly. For that reason, it is more useful to use d1 than, for example, d5. It is thus more appropriate to use short waves than long waves to

https://doi.org/10.7441/joc.2023.01.04

detect changes. The Fourier transform is unable to detect the moment of change, while the wavelet transform makes this moment obvious.

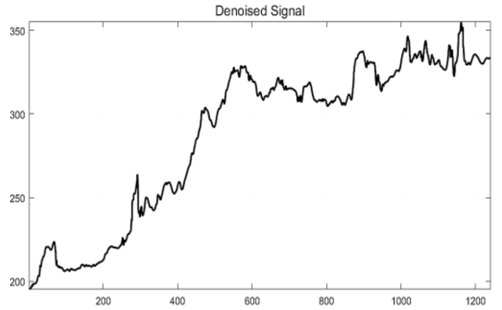

Fig. 10 – Wavelet denoising of SAX index. Source: own research.

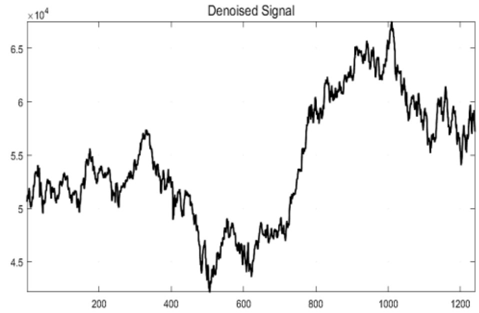

Fig. 11 – Wavelet denoising of WIG index. Source: own research.

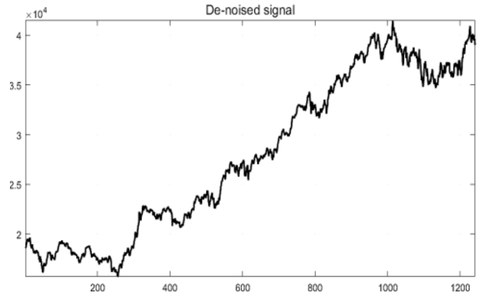

Fig. 12 – Wavelet denoising of BUX index**.** Source: own research.

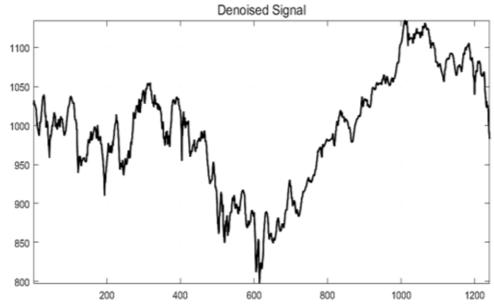

Fig. 13 – Wavelet denoising of PX index. Source: own research.

Denoising using wavelet analysis can remove noise in a time series while maintaining sharp parts for its excellent time-frequency localization feature. Figures 10–13 illustrate the noise indexing of stock indices. As Lai & Huang (2007) describe, this is the simplest and most effective approach to removing the noise of a wavelet transform.

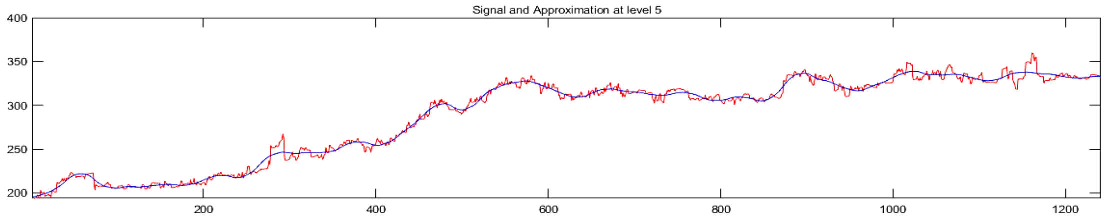

Fig. 14 – Wavelet approximation of SAX index. Source: own research.

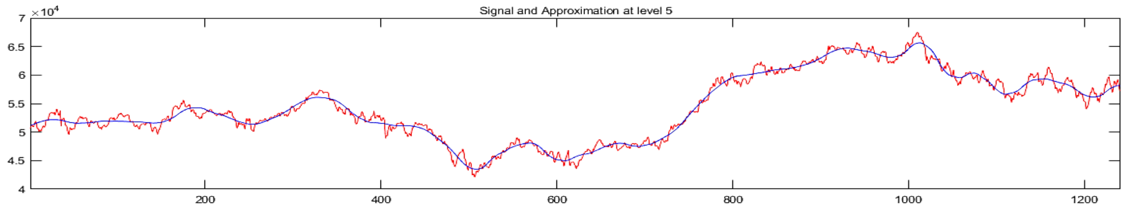

Fig. 15 – Wavelet approximation of WIG index. Source: own research.

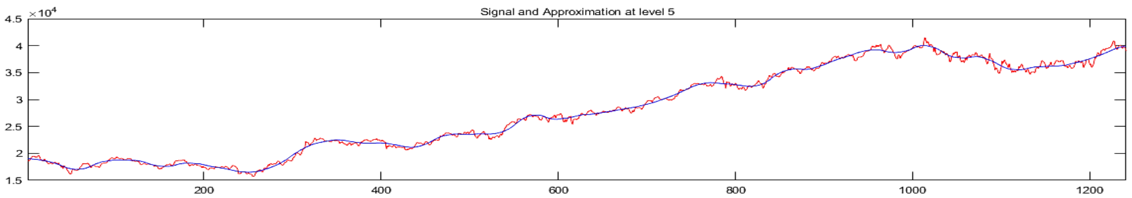

Fig. 16 – Wavelet approximation of BUX index. Source: own research.

https://doi.org/10.7441/joc.2023.01.04

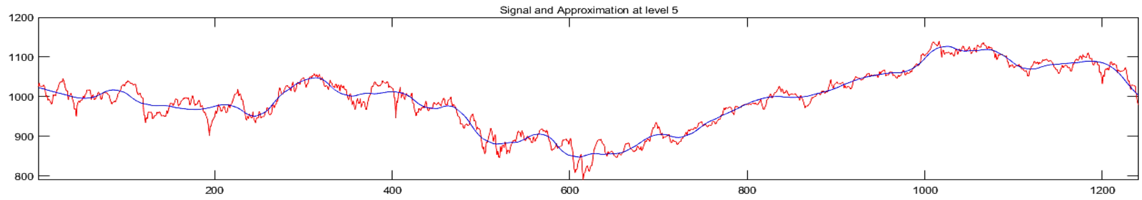

Fig. 17 – Wavelet approximation of PX index. Source: own research.

With each subsequent approximation, the trend of the time series appears more and more distinct and smoother, as it corresponds to the slowest part of the signal of the monitored stock indices. As the value of the scale decreases with resolution, this leads to a better estimate of the trend of stock indices. It can be noticed that the highest frequency appears at the beginning of the original time series. With each new approach, the time series appears less noisy but loses more high-frequency information. For example, when approximating the time series labeled a5, approximately, the first 20% of the original time series is shortened. In addition, if the time series contains many sharp changes, each additional approximation looks less similar to the original time series.

4.3. Forecasting Using Neuro-Fuzzy Approach

The daily closing prices of stock indices for the observed time period of 2014 to 2018 are used for the prediction. The analyzed data are pre-processed to avoid skewing the results, as the stock index values are significantly different, which could affect an overall evaluation of the WANFIS model:

$$y_t = \frac{x_t - m}{M - m} \tag{10}$$

where x_t is the closing daily price of the time series at time t, $M = max\{x_t\}$ and $m = min\{x_t\}$. The variables entering the expert model are subsequently defined as:

$$(y_{t-3}, y_{t-2}, y_{t-1}, y_t) \tag{11}$$

First, the initial structure of the ANFIS model is generated using a grid partitioning technique. As described by Talpur et al. (2017), this technique generates a FIS model of the Sugeno type using a training data set and according to the selected number of membership functions and the selected type. The presented model has three inputs and one output, which indicates the predicted value. This predicted value indicates the normalized price of 4 stock indices of the Visegrad countries. Due to the character of data from stock markets, the Gaussian membership function is chosen, as it provides the highest accuracy of prediction, as also confirmed by the research by Talpur et al. (2017). The input variables are further divided into three attributes, which are represented by three membership functions. The generated ANFIS architecture is based on the grid division method. Here, not only the inputs and functions of the membership are graphically captured, but also the interconnection of the rules base, which is generated using artificial neural networks. The created ANFIS model contains 27 rules generated through a neural network. One membership output function is specified for each generated rule. A hybrid optimization method is chosen for the ANFIS model training. The combination of least squares method and gradient descent method results in this method. During the training process, training data in cycles or epochs with forward and backward passages are presented to the model. Within the created model, 30 training epochs are selected and the error tolerance is set to zero due to the uncertainty of errors during the training process. This framework is recommended in the literature (Azar, 2010). The results of the predicted values of the stock indices through the smoothed time series by wavelet analysis and the original time series are measured using the RMSE indicator, which compares the original data y_t and the obtained data $\dot{y}_t$. Calculation of the RMSE indicator is possible according to the following formula:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t - \dot{y}_t)^2} \tag{12}$$

The RMSE indicator of the training and test data for selected stock indices for WANFIS and ANFIS is shown in Table 3. It is obvious that the WANFIS model achieves a lower error rate, where RMSE is very low. The smallest training data error is manifested for SAX, and PX with error values of 0.03% and 0.041%, respectively. BUX, on the other hand, generated the highest error with a value of 0.078%. The most accurate prediction from the point of view of the ANFIS model in the training period with an error value of 0.014% is for the Slovakia SAX index; on the contrary, the highest errors were achieved by BUX with 0.075% and WIG with 0.035%. The lower training values are certainly due to a small sample of training data. However, despite this, the RMSE is very low for all V4 stock indices due to the wavelet transformation and the use of modified wavelet denoising data that enter the created neuro-fuzzy model.

Tab. 3 – Training and testing RMS error. Source: own research.

Stock index	WANFIS		ANFIS	
	Training error	Testing error	Training error	Testing error
SAX	0.00030	0.00014	0.01868	0.02247
WIG	0.00047	0.00035	0.01865	0.01944
BUX	0.00078	0.00075	0.01126	0.00829
PX	0.00041	0.00024	0.02192	0.02411

From the above results, it can be summarized that both the ANFIS and WANFIS models show a lower error rate for less liquid stock indices in the training data set, while more efficient and more liquid stock indices are more accurate for the test data set. However, the error rate in both sets is very low. The WANFIS model generally shows a significant improvement and a lower error rate in terms of the RMSE indicator than the classic ANFIS. It can be said that the WANFIS model is an effective tool for predicting the development of stock markets in Central European countries.

5 CONCLUSION

The paper deals with a hybrid approach using the wavelet transformation and a neuro-fuzzy approach (WANFIS) for predicting the development of selected stock indices of Central European countries. The stock markets of the Czech Republic, Slovakia, Poland and Hungary, referred to as the Visegrad 4 countries, are atypical markets that have not yet been explored and analyzed in terms of the WANFIS hybrid system. In the available literature (Sharma et al., 2021), the prediction deals exclusively with developed markets such as the American stock market or selected stocks (Kumar Chandar, 2019; Alenezy et al., 2021) and the less liquid and less efficient European stock markets are completely neglected. For this reason, the paper focuses on these markets as an extension of existing knowledge. The development of an effective tool in the financial system can fundamentally increase the competitive advantage not only of investors but also of companies in market environments. Wavelet analysis brings a new vision of signal processing. Tactically, it avoids the problem encountered in the Fourier analysis. Decomposition of the financial time series using wavelet transform analyzes the signal of continuous development from a different perspective. The advantage is the possibility of more accurate localization of time and frequency. After stock index smoothing, the state of knowledge about wavelet transformation is extended by the prediction using the hybrid neuro-fuzzy inference system. The fuzzy approach of the Sugeno type is chosen for the prediction of development, on the basis of which Gaussian fuzzy membership functions are defined. Subsequently, the knowledge base of rules is defined by means of neural networks, by which the future development of V4 stock markets is determined. The results show that the proposed hybrid WANFIS model demonstrates a more accurate prediction of the development of stock indices than the individual models alone. Experimental results show that the fusion model provides a promising and effective tool for predicting even less liquid and less efficient stock markets such as those in the V4 countries. The results show a lower error rate of the new model and provide impressive results that can help investors make investment decisions. A useful and accurate prediction alternative proven in emerging stock markets is offered. In subsequent research, it may be appropriate to modify the WANFIS model and use type-2 fuzzy membership functions instead of type-1 fuzzy membership functions uncertainties and are able to overcome the classical fuzzy logic, as noted for example by Janková et al. (2021). Furthermore, it is necessary to examine the various functions of membership in the fuzzy model to see if they are able to provide more accurate results.

ACKNOWLEDGEMENT
This paper was supported by the project No. FP-S-22-7977 'Modeling and optimization of processes in the corporate sphere' of the Internal Grant Agency at the Brno University of Technology.

References

1. Alenezy, A. H., Ismail, M. T., Wadi, S. A., Tahir, M., Hamadneh, N. N., Jaber, J. J., & Khan, W. A. (2021). Forecasting stock market volatility using hybrid of adaptive network of fuzzy inference system and wavelet functions. *Journal of Mathematics (Hidawi)*, (2021), 9954341. https://doi.org/10.1155/2021/9954341
2. Aloui, C., Hkiri, B., Lau, M. C. K., & Yarovaya, L. (2018). Information transmission across stock indices and stock index futures: International evidence using wavelet framework. *Research in International Business and Finance*, *44*, 411–421. https://doi.org/10.1016/j.ribaf.2017.07.112
3. Artha, S. E. M., Yasin, H., Warsito, B., & Santoso, R. (2018). Application of Wavelet Neuro-Fuzzy System (WNFS) method for stock forecasting. *Journal of Physics: Conference Series*, *1025*(1), 12101. https://doi.org/10.1088/1742-6596/1025/1/012101
4. Azar, A. T. (2010). *Fuzzy Systems*. IN-TECH.
5. Budapest Stock Exchange (BSE). (2020). *Market overview, equity indices*. https://bse.hu/
6. Bratislava Stock Exchange (BSSE). (2020). *Index SAX*. http://www.bsse.sk/
7. Dash, S. R., & Maitra, D. (2018). Does sentiment matter for stock returns? Evidence from Indian stock market using wavelet approach. *Finance Research Letters*, *26*, 32–39. https://doi.org/10.1016/j.frl.2017.11.008
8. Dima, B., Dima, Ş. M., & Barna, F. (2015). A wavelet analysis of capital markets' integration in Latin America. *Applied Economics*, *47*(10), 1019–1036. https://doi.org/10.1080/00036846.2014.987917
9. Forsyth, J. A., & Mongrut, S. (2022). Does duration of competitive advantage drive long-term returns in the stock market? *Revista Contabilidade & Finanças*, *33*(89), 329–342. https://doi.org/10.1590/1808-057x202113660
10. Polish Capital Market (GPW). (2020). *Index facktsheet*. https://www.gpw.pl
11. Jang, J.-S. R. (1993). ANFIS: Adaptive-network-based fuzzy inference system. *IEEE Transactions on Systems Man, and Cybernetics*, *23*(3), 665–685.
12. Jang, J.-S. R. (1992). Self-learning fuzzy controllers based on temporal backpropagation. *IEEE Transactions on Neural Networks*, *3*(5), 714–723. https://doi.org/10.1109/72.159060

13. Janková, Z., & Rakovská, E. (2022). Comparison uncertainty of different types of membership functions in T2FLS: Case of international financial market. *Applied Sciences*, *12*(2), 918. https://doi.org/10.3390/app12020918
14. Janková, Z., Jana, D. K., & Dostál, P. (2021). Investment decision support based on interval type-2 fuzzy expert system. *Inžinerinė Ekonomika*, *32*(2), 118–129. https://doi.org/10.5755/j01.ee.32.2.24884
15. Janková, Z. (2020). Sentiment on the stock markets: Evidence from the wavelet coherence analysis. *Scientific Papers of the University of Pardubice*, Series D, *28*(3), 1–10.
16. Kristjanpoller, W. R., & Michell, K. (2018). A stock market risk forecasting model through integration of switching regime, ANFIS and GARCH techniques. *Applied Soft Computing*, *67*, 106–116. https://doi.org/10.1016/j.asoc.2018.02.055
17. Kumar Chandar, S. (2019). Fusion model of wavelet transform and adaptive neuro fuzzy inference system for stock market prediction. *Journal of Ambient Intelligence and Humanized Computing*, (2019). https://doi.org/10.1007/s12652-019-01224-2
18. Lai, L, & Liu, J. (2014). Support vector machine and least square support vector machine stock forecasting models. *Computer Science and Information Technology*, *2*(1), 30–39.
19. Lai, K. K., & Huang, J. (2007). The application of wavelet transform in stock market. In Y. Nakamori (Ed.), *Proceedings of the Eighth International Symposium on Knowledge and Systems Sciences* (pp. 59–66). JAIST Press.
20. Li, Z., & Tam, V. (2017). Combining the real-time wavelet denoising and long-short-term-memory neural network for predicting stock indexes. In *2017 IEEE Symposium Series on Computational Intelligence (SSCI)* (pp. 1–8). IEEE Service Center. https://doi.org/10.1109/SSCI.2017.8280883
21. Lin, F.-L., Yang, S.-Y., Marsh, T., & Chen, Y.-F.. (2018). Stock and bond return relations and stock market uncertainty: Evidence from wavelet analysis. *International Review of Economics & Finance*, *55*, 285–294. https://doi.org/10.1016/j.iref.2017.07.013
22. Livan, G., Inoue, J., & Scalas, E. (2012). On the non-stationarity of financial time series: Impact on optimal portfolio selection. *Journal of Statistical Mechanics*, *2012*(7). https://doi.org/10.1088/1742-5468/2012/07/P07025
23. Mathur, N., Glesk, I., & Buis, A. (2016). Comparison of adaptive neuro-fuzzy inference system (ANFIS) and Gaussian processes for machine learning (GPML) algorithms for the prediction of skin temperature in lower limb prostheses. *Medical Engineering & Physics*, *38*(10), 1083–1089. https://doi.org/10.1016/j.medengphy.2016.07.003
24. Pabuçcu, H. N., & Değirmenci, N. (2018). Volatilitenin modellenmesi ve anfis model ile bist100 getiri tahmini. adam akademi. *Sosyal Bilimler Dergisi*, *8*(2), 367–387.
25. Pinho, C., & Madaleno, M. (2010). Time frequency effects on market indices: World comovements. *China-USA Business Review*, *9*(4),1–24.
26. Polanco-Martínez, J. M., Fernández-Macho, J., Neumann, M. B., & Faria, S. H. (2018). A pre-crisis vs. crisis analysis of peripheral EU stock markets by means of wavelet transform and a nonlinear causality test. *Physica A*, *490*, 1211–1227. https://doi.org/10.1016/j.physa.2017.08.065
27. Prague Stock Exchange (PSE). (2020). *Hodnoty indexu PX*. https://www.pse.cz/indexy
28. Rajab, S., & Sharma, V. (2019). An interpretable neuro-fuzzy approach to stock price forecasting. *Soft Computing*, *23*(3), 921–936. https://doi.org/10.1007/s00500-017-2800-7

29. Ramsey, J. B. (1999). Regression over timescale decompositions: A sampling analysis of distributional properties. *Economic Systems Research*, *11*(2), 163–184. https://doi.org/10.1080/09535319900000012
30. Raoofi, A., & Mohammadi, T. (2018). Forecasting Tehran Stock Exchange Index returns using a combination of wavelet decomposition and adaptive neural fuzzy inference systems. *Allameh Tabataba'i University Press*, *23*(76), 107–136.
31. Rua, A. (2012). Money growth and inflation in the Euro Area: A time-frequency view. *Oxford Bulletin of Economics and Statistics*, *74*(6), 875–885. https://doi.org/10.1111/j.1468-0084.2011.00680.x
32. Rua, A., & Nunes, L. C. (2009). International comovement of stock market returns: A wavelet analysis. *Journal of Empirical Finance*, *16*(4), 632–639. https://doi.org/10.1016/j.jempfin.2009.02.002
33. Şahin, M., & Erol, R. (2017). A comparative study of neural networks and ANFIS for forecasting attendance rate of soccer games. *Mathematical and Computational Applications*, *22*(4), 43. https://doi.org/10.3390/mca22040043
34. Sharma, D. K., Hota, H. S., & Rababaah, A. R. (2021). Forecasting US stock price using hybrid of wavelet transforms and adaptive neuro fuzzy inference system. *International Journal of System Assurance Engineering and Management*, *12*(4). https://doi.org/10.1007/s13198-021-01217-5
35. Talpur, N., Salleh, M. N. M., & Hussain, K. (2017). An investigation of membership functions on performance of ANFIS for solving classification problems. *IOP Conference Series: Materials Science and Engineering*, *226*(1), 12103. https://doi.org/10.1088/1757-899X/226/1/012103
36. Tiwari, A. K., Mutascu, M. I., & Albulescu, C. T. (2016). Continuous wavelet transform and rolling correlation of European stock markets. *International Review of Economics & Finance*, *42*, 237–256. https://doi.org/10.1016/j.iref.2015.12.002
37. Xu, W.-J., & Zhong, L.-X. (2022). Market impact shapes competitive advantage of investment strategies in financial markets. *PloS One*, *17*(2), e0260373-e0260373. https://doi.org/10.1371/journal.pone.0260373
38. Xu, Y., Liu, Z., Zhao, J., & Su, C. (2017). Weibo sentiments and stock return: A time-frequency view. *PloS One*, *12*(7), e0180723-e0180723. https://doi.org/10.1371/journal.pone.0180723
39. Zarei, G., Mohamadiyan, R., & Hazeri, H. (2018). The comparison of fuzzy neural network methods with wavelet fuzzy neural network in predicting stock prices of banks accepted in Tehran Stock Exchange. *Financial Management Strategy*, *6*(3), 109–138. https://doi.org/10.22051/jfm.2018.19214.1606

Contact information

Zuzana Janková, Ph.D.
Brno University of Technology
Faculty of Business and Management
Institute of Informatics
Czech Republic
E-mail: Zuzana.Jankova@vutbr.cz
ORCID: 0000-0003-1798-5275