Investment Portfolio Evaluation by the Fuzzy Approach

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Abstract

This paper presents a new fuzzy approach for the evaluation of investment portfolio, where the approach is viewed by the authors as a sub-phase of the management process of these portfolios. The approach defines the mutual and delayed effects among the significant variables of the investment portfolio. The evaluation of the effects is described as fuzzy trapezoidal numbers and they are aggregated by mathematical operations with incidence matrices and fuzzy functions "experton".

Key words: management process of investment portfolio, fuzzy evaluation; fuzzy expertons and incidence matrices; delayed effects

1. INTRODUCTION

Portfolio management is a well-researched interdisciplinary field. At the same time, there are many new possibilities for innovation through application of various new methods for solving the problem. Fuzzy logic and fuzzy sets are increasingly popular in portfolio management.

The main focus of this paper is on proposing a new fuzzy approach for evaluating investment portfolios. This aim is achieved by consequently fulfilling several research tasks. First is to review the general concept of investment portfolio and the process of investment portfolio management. Next is to point out possible fuzzy approaches for portfolio management. Then are defined the terminology of the used methods and stages of the portfolio evaluation. Finally an empirical approbation is conducted.

In methodical terms the approach suggested uses tools of the theory of confidence intervals, theory of fuzzy subsets and method of expertise. A range of fuzzy instruments are used –fuzzy trapezoidal numbers, fuzzy functions "experton" and fuzzy random incidence matrices.

The paper is divided into six chapters, corresponding to the structure of a scientific article. Some important terms of the portfolio theory are defined in sub-chapter 2.1. Process of the portfolio management is describes and dissects in sub-chapter 2.2. Known and possible fuzzy approaches to portfolio management are reviewed in sub-chapter 2.3. Sub-chapter 3.1 outlines the concept of the current proposal. Tools necessary for the proposal fulfillment are described in sub-chapter 3.2. Stages of the approach suggested are presents in sub-chapter 3.3. Results of the approach approbation are presented in chapter 4, followed by short discussion (chapter 5) and conclusion (chapter 6).

2. THEORETICAL SOLUTIONS

2.1 General outline of the investment portfolio

The investment portfolio is a combination of securities owned by an investor. Securities are investment opportunities (investment assets), traded freely on a transparent market. Transparent market is one publicly transmits enough relevant information. The most important feature of the securities is that they are interdependent – their prices covariate reflecting the real-world interdependence of the issuing companies.

The purpose of using a portfolio approach is to improve the conditions of the investment process by obtaining such investment properties of the combination of securities, which are not obtainable by any single security. The most rottenly considered significant variables for any investment are risk and return. A certain configuration of risk and return is only possible within a given configuration of securities. Diversification is the effect of combining multiple securities such that it improves the risk conditions of investment.

A portfolio consists of *k+1* positions each with respective weights, where k is the total number of positions traded on the market (formula 1). The invested sum of unwanted positions is set to 0. The non-invested amount is assumed a cash position *C*. If the cash position is less than 0, then there are borrowed funds. Short positions are also possible, in which case the sum invested in security *i* is negative.

$$(1) \quad P(t) = \sum_{i=1}^{k} s_i(t) + C(t)$$

where:

i	- serial number of position;
k	- total number of possible non-cash positions;
t	- time of observation;
$P(t)$	- value of the portfolio at the time t;
$s_i(t)$	- allocated investment of position i at the time t;
$C(t)$	- value of cash position at the time t.

Return of an investment portfolio (see formula 2) is calculated as a weighted average of the returns of all included securities. The weights correspond to the configuration of the portfolio – the allocated investment in each position. The sum of all weights (including cash position) is always equal to 1. The return of a cash position is normally assumed 0.

$$(2) \quad R_p(t) = \sum_{i=1}^{k} w_i(t).R_i(t)$$

where:

$R_p(t)$	- return of the portfolio p at the time t;
$R_i(t)$	- return of the security i at the time t;
$w_i(t)$	- relative weight of position i at the time t.

The first assumption to calculate return of each security at the time $t(R_i(t))$ is that the time domain is discrete. The return Also should account for market frictions, such as taxes, brokerages, inflation rate, etc. The most general case is shown in formula 3, with some further elaboration in formulas 4, 5.

$$(3)\quad R_i(t) = \frac{RP_i(t).\left(1-D_p\right)+RC_i(t).\left(1-D_c\right)}{P_i(b)} I(b,s); D_p, D_c \in [0,1]$$

where:

$RP_i(t)$ - return from capital gains of security i at the time t ;

$RC_i(t)$ - return from complimentary benefits of security i at the time t ;

D_p - function for tax rate on capital gains;

D_c - function for tax rate on complimentary benefits;

$P_i(b)$ - buy price at the time b of security i ;

$I(b,s)$ - inflation rate between time b and s ;

$$(4)\quad RP_i(t) = P_i(s).h_i - P_i(b) - K(s) - K(b); h_i \begin{cases} >1, b>s \\ <1, b<s \end{cases}$$

where:

$P_i(s)$ - sell price at the time s of security i ;

$P_i(b)$ - buy price at the time b of security i ;

h_i - stock split correction coefficient of security i ;

$K(s)$ - brokerage at the time s ;

$K(b)$ - brokerage at the time b .

$$(5)\quad RC_i(t) = \sum_{t=\min(b,s)}^{\max(b,s)} B_i(t)$$

where:

$B_i(t)$ - quantified complimentary benefits of security i at time t

Formula 4 accounts for the possibility of short selling: it is not known which of the times *s* and *b* precedes the other. It also accounts for brokerages. To take account of possible stock split operations during the period of investing in the security, a correction coefficient *h* is introduced. Depending on the type of position – long or short – the value of h could be: $h>1$ for long positions i.e. $x/1$, where x is the stock split ratio or $h<1$ for short positions i.e. $1/x$, where x is the stock split ratio

Formula 5 accounts for the fact that besides the return derived from price change, there are other forms of return of a security emerging during the time of investing. These include dividends, interest, non financial benefits and etc. All these must be estimated as financial inflow or outflow per one share (e.g. if while holding a short position there is a dividend of z amount per share, then this is a negative return of *z*).

There are several approaches to calculating portfolio risk. The dominant concept is to use historical variance and/or standard deviation. Other rottenly used measures are historical volatility and value at risk. A good case could be build around using information entropy as a risk measure of a portfolio. So measuring the risk of an individual security may be formulated as function of historical data of the return of security (see formula 6):

$$(6)\quad V_i(t) = F\left(\left(\overrightarrow{t-d}\right), t\right)$$

where:

$V_i(t)$ - risk of the security i at the time t;

F - function for measuring the risk of security i;

d - number (depth) of historical data considered for calculation of risk.

No matter what measure is used the risk of a portfolio depends not only on the risks of every included security, but also on the mutual dependence (interdependence) among the securities, except for cash position, which is assumed to have a risk of 0. A typical approach to measure portfolio risk is a sum of two addends – one for weighted average of the risks of included securities and the other for calculating the interdependence of the securities (Formula 7).

$$(7)\quad V_p(t) = \sum_{i=1}^{k} w_i(t)^2 . V_i(t) + \sum_{i=1}^{k}\sum_{j=1}^{k} w_i(t).w_j(t).\rho\left(V_i(t), V_j(t)\right)$$

where:

$V_p(t)$ - risk of the portfolio p at the time t;

$V_i(t)$ - risk of the security i at the time t;

$\rho\left(V_i(t), V_j(t)\right)$ - measure for interdependence of the securities i and j.

2.2 Phases in the process of portfolio management

The process of portfolio management could be analyzed in several phases that are arranged within a control cycle. At the same time portfolio management is an information transforming process. As such it may be analyzed as consisting of three general phases which could be dissected further into functional sub-phases, as follows:

1. Information input – In this phase the ingoing informational flow is encoded in an understandable form.

1.1. Setting goals – A goal is a desired state (configuration) of the significant variables. After the first controlling cycle, an additional task is included in goal setting – comparing the current state with the desired one. Criteria for evaluating portfolio performance may be used. Very suitable for the task is the Sortino ratio or its modification. The ratio is naturally goal oriented as it compares the achieved return with a desired return.

1.2. Receiving, collecting, systemizing information on the behavior and the structure of the portfolio. - This sub-phase closes the feedback loop of the controlled process.

1.3. Receiving, collecting, systemizing information about market (environment) – This subphase works with information from the known, observed external factors (market conditions and constraints, obtainable investment opportunities), influencing the portfolio management process.

2. Information processing – This phase is associated with making the best possible use of the information obtained according to the needed function of portfolio management.

2.1. Forecasting / estimating the expected values of the significant variables of the obtainable investment opportunities and the external factors. Statistical analysis of the past portfolio structure is also necessary.

2.2. Solution generation – This is the process of defining and evaluating feasible states of the portfolio as combinations of multiple securities. There is a necessity of having an external model to simulate possible solutions to the portfolio problem. It is not a compulsory component but using an example model („étalon”) is normal in investment portfolio management. It is a computerized simulation model for experimenting and evaluating the generated solutions. In most cases, the computer simulation would be programmed along a known (or new) theory (for instance Markowitz Model).

2.3. Making decision and selecting a portfolio structure. Only "optimal" (best possible) solutions out of all feasible are considered. There is a need for using multi-criteria optimization and enforcing the principle of requisite addition. An important variable to be considered is the investor's rationality and his preferences towards risk (and towards other significant variables).

3. Information output – This stage is associated with the transmission (decoding) the information necessary for the management effects of the portfolio At this phase the controlling actions are emitted toward the portfolio, which also means realization of the solution. After comparison between the desired structure and the current structure of the portfolio, the differences are translated into market orders. Several real limitations interfere with the realization of the solution and thus the real implementation is always sub-optimal:

- Discretization, dissectability, availability of an issue of a given security – The numerical problem becomes a whole number optimization problem.
- Delay of the system reaction, including the time for executing an order, as well the time for meeting the conditions of the order. The inertness of the controlled system also enforces delays.
- Market friction is the cumulative effect on the free trade from brokerages, the inflation rate of the economy, taxes on capital gains and/or dividends/interests, etc.

2.3 Fuzzy approaches to the portfolio problem

Since the decision for a portfolio structure relies on ex-ante estimation based on ex-post data, the process is carried out under uncertainty generated by the unknown future outcomes (Marcheva, 1995). Furthermore the huge complexity and abnormality (Markowitz, Usmen, 1996, p. 22) of the financial markets makes the stochastic (let alone the deterministic) approach less and less applicable, because there is no base for assuming any given probability distribution of the security return. So other approaches to deal with the uncertainty of the portfolio are being sought by the researching community.

There is an ongoing discussion on how to define uncertainty in the context of fuzzy approach to portfolio management. A very systematic and tidy analysis is done by Zmeskal (2005). Uncertainty is a twofold meaning term. First uncertainty stands for measured uncertainty – risk. When dealing with classical definitions of risk – it is either defined in deterministic or in stochastic terms, resulting in a crisp (as opposed to fuzzy) set of numbers and crisp values. The other way of defining uncertainty is vagueness – the unmeasured uncertainty.

When discussing portfolio management both meanings of uncertainty are considered. First some sort of measurement is needed, but as well as a method to compensate the impreciseness of such method.

A possible tool for the task is the fuzzy approach i.e. using fuzzy numbers and fuzzy sets to describe uncertain phenomena and/or using fuzzy logic to process data from uncertain phenomena. A complete fuzzy approach for portfolio management would be a fuzzy control process entirely made of fuzzy sub-phases:

- Fuzzy information input – fuzzification of data from the portfolio and the environment. As for the goal setting sub-phase, the goals originate as linguistic variables anyway. So it is just a matter to make them compatible with the rest of the process in information terms.
- Fuzzy information processing would mostly use fuzzy logic and fuzzy mathematics. There are already a lot of proposals of this type to estimate the significant variables and generate solutions (see below). Some of them even suggest ways of fuzzy selection and evaluation of solutions by fuzzy functions.
- Fuzzy information output would be the phase to conduct defuzzification of the solution and to carry out management actions on the portfolio.

Once the fuzzy approach for solving problems under conditions of uncertainty is becoming increasingly popular among researchers, it is quite expected that there is already a wide range of proposed solutions for different phases and/or tasks of the process of portfolio management. The propositions are most often oriented towards the two more technical phases of portfolio management:

Fuzzy approaches to estimating significant variables of a portfolio

This is the most common suggestion for using fuzzy approach in portfolio management. The authors propose fuzzy measures of return and risk of the portfolio. Typically they are followed by a way to estimate the variance – covariance matrix necessary for portfolio optimization. Good examples are the works of Katagiri and Ishii (1999), Mohamed et al. (2009), Petreska and Kolemisevska (2010) and Zhang et al. (2003). Fuzzy membership functions are used to adjust the return and the risk of the securities in the study of Lian and Li (2010). The portfolio risk measure is a fuzzy estimated type of value at risk in the studies of Liu et al. (2005) and Wang et al. (2009). An unorthodox measure of portfolio risk is proposed by Huang (2008) – the entropy of fuzzy returns of the securities in the portfolio.

An interesting and somewhat related to the proposition in the current paper is the approach of Tastle and Wierman (2009). The authors there use expert opinions to reach a degree of consensus on risk estimation. Also similar to some extend is the study of Marcheva (1995). It is another research using interval numbers, where forecasting of shares prices is done by experts.

Fuzzy approaches for generating feasible solutions to a portfolio problem

Authors focus on using fuzzy reasoning i.e. fuzzy subsets, fuzzy rules and linguistic variables for selecting portfolio structure or realization of investment strategy. In his classical book Bojadziev and Bojadziev (1997, pp. 157-164) propose such approach for one of the first times. Later Chow and Inoue (2001) Ghandar et al. (2009) and Nakaoka et al. (2005) elaborate on fuzzy linguistic rules.

A fuzzy ranking strategy for portfolio selection giving "best solutions" for different degrees of risk-aversion is proposed by Bermudez et al. (2007). And Tiryaki and Ahlatcioglu (2009) use a fuzzy analytical hierarchical approach for multi-criteria selection of securities in a portfolio.

3. METHODOLOGY

3.1 The proposed fuzzy approach for portfolio evaluation

Current paper proposes a fuzzy approach for evaluating a portfolio structure using expertise. An important remark that has to be made upfront is that the term expertise is used in a broad sense. So an expert evaluation may represent the computation from a mathematical algorithm, a statement form a person with special and extended knowledge on the subject or combination of both.

The process of evaluation of the portfolio begins after a portfolio structure has been already set. Second stage uses experts' evaluations or evaluations from mathematical algorithms (called method of expertise hereafter), presented in the form of fuzzy trapezoidal numbers. The fuzzy trapezoidal numbers have membership function which specifically displays a maximum range (instead of a point) of values among the values of the estimated variable.

The fuzzy numbers are then processed in a specific method for discovering the influences of return on risk among the securities and within the portfolio. Analysis on delayed influences is later done.

The aim of the approach is to establish a method for evaluating investment portfolios by determining the mutual influences among different significant variables of the portfolio (in that case, return and risk) and the hidden influences between them. The approach suggested could also be used as a base for comparison and/or ranking different portfolios. Last but not least the used experts' evaluations may be aggregated results from other approaches for portfolio management. Thus, the approach could be described as a universal tool to combine several methods, while averaging out their extreme solutions.

3.2 Tools for portfolio evaluation

Portfolio evaluation finds expression in two activities in this approach. The first activity is evaluation of the return influence on the risk of shares in the portfolio taking into account mutual influences between returns of shares and between their risks. The second activity is evaluation of delayed effects of returns on risks of shares in the portfolio.

Tools, suggested in the paper, for the portfolio evaluation consists of:

- method of expertise;
- mathematical operations with confidence intervals with four evaluations ("confidence fours"); and
- mathematical operations with fuzzy trapezoidal numbers (FTNs), fuzzy expertons, fuzzy random incidence matrices.

Method of expertise is used to evaluate returns and risks of shares in the portfolio as well as the influence of returns on risks of shares. The evaluations are systematized in fuzzy matrices of: influence of returns on risks, mutual influences between returns of shares and mutual influences between risks of shares. Possible interval of change [*0,1*] is set for the evaluations. The method of expertise is applied due to the authors' belief in low utility of statistical methods for evaluating under uncertainty.

Confidence intervals with four evaluations are a tool of the theory of intervals. It is a branch of mathematics applied to conditions of subjectivity and uncertainty (Kaufmann, Gil Aluja, 1990, p. 11). According to the theory the evaluation is described by an interval, which is not characterized by a possibility of occurring and convexity (Kaufmann, Gil Aluja, 1990, p. 21). In this context confidence fours are building elements of fuzzy random incidence matrices and functions "experton" in the aggregation procedure of the portfolio evaluations. In this approach confidence fours are presented in discrete form (defuzzificated) by so-called "representative number of the confidence four". It reflects the relative linear distance of the interval to the number "zero" on the explicit condition of absent possibility of occurring (Kaufmann, Gil Aluja, 1988, p. 74). Representative numbers are used in the approach to define delayed effects between returns and risks as well as to present results of the portfolio evaluation more clearly.

Three types of tools of the theory of fuzzy subsets are used in the approach. The first one is fuzzy subset/number. It is described by confidence intervals for any possibility of occurring in the interval [*0,1*] (Kaufmann, Gil Aluja, 1986, p. 37). Fuzzy trapezoidal numbers are used to describe uncertain experts' evaluations of influences of: returns on risks of the shares in the portfolio, returns between shares and risks between them. The fuzzy trapezoidal number is a fuzzy number/subset with a linear and continuous characteristic function, which has two evaluations of possibility of occurring "unity" and two evaluations of possibility of occurring "zero" (Bojadziev, Bojadziev, 1997, pp. 24-25).

Mathematical operations with fuzzy random incidence matrices (Kaufmann, Gil Aluja, 1988, p. 54) are used to aggregate evaluations of influences and to study combined and delayed effects between returns and risks. Three operations with fuzzy random matrices are used in the approach – "maxmin" function, calculation of the mathematical expectation of matrices and difference between matrices. The "maxmin" function is applied for evaluation of combined influences of I and II generations of returns on risks (formula 8). The mathematical expectation weighs the evaluations of influences against the possibilities of their occurring. It is used as a basis for determining the delayed effects of returns on risks.

Fuzzy functions "experton" are kind of fuzzy random matrices. They are used in the approach to aggregate the evaluations. The experton function is defined as a matrix describing the law on cumulative (for all experts) complementary (in this case to the number "unity") probable distribution of evaluations (Kaufmann, Gil Aluja, 1990, p. 54-55).

3.3 Stages of portfolio evaluation

According to the authors' idea the portfolio evaluation could be implemented in four stages:

- Stage I "Determining the portfolio";
- Stage II "Aggregation of evaluations of (mutual) influences between return and risk of shares in the portfolio";
- Stage III "Evaluation of combined influences (of I and II generations) of returns on risks of shares in the portfolio"; and

- Stage IV “Evaluation of delayed effects of returns on risks of shares in the portfolio”.

The first stage consists of procedures for portfolio generating and evaluation of (mutual) influences of returns and risks of the shares in the portfolio. The first procedure is not subject to this publication. The second procedure covers activities of generating matrices of (mutual) influence of returns and risks in the portfolio, including matrices of: influence of returns on risks of the shares in the portfolio, mutual influence between returns of the shares and mutual influence between risks of the shares. In mathematical terms the evaluations are represented by fuzzy trapezoidal numbers.

The evaluations of influence of returns and risks of the shares in the portfolio are aggregated at the second stage. This is achieved by forming experton functions, which require use of fuzzy trapezoidal numbers as confidence fours. The second stage consists of the following procedures:

- calculation of experton of mutual influences between returns of the shares;
- calculation of experton of mutual influences between risks of the shares; and
- calculation of experton of influence of returns on risks of the shares.

Mutual influences between returns of shares in the portfolio are aggregated in the first procedure. The procedure includes accumulation of the evaluations of mutual influence between returns of the shares by fuzzy random influence matrices and formation of the experton of mutual influences between returns of the shares. Mutual influences between risks of the shares in the portfolio are aggregated in the second procedure. Influences of returns of the shares on their risks are aggregated in the third procedure. The second and third procedures are realized in analogy with the first procedure of the stage.

Combined influences of I and II generations of returns on risks of the shares in the portfolio are evaluated at the third stage. It is implemented by integrating mutual influences between returns of the shares, risks of the shares and influence of returns on risks into so-called “combined influences of I and II generations”. Combined influences are evaluated by applying the function “maxmin” to the expertons: “return - return”, “risk - risk” and “return – risk” (formula (8)).

$$(8)\quad \ddot{\tilde{I}}_{I,II} = \dot{\tilde{Y}} \circ \dot{\tilde{E}}_{Y\to R} \circ \dot{\tilde{R}} = \vee\left(\dot{\tilde{Y}} \wedge \dot{\tilde{E}}_{Y\to R} \wedge \dot{\tilde{R}}\right)$$

where:

$\ddot{\tilde{I}}_{I,II}$ - is fuzzy random matrix of combined influences of I and II generations;

$\circ$ $\vee$, $\wedge$ - are symbols to denote functions “maxmin”, “max” and “min” respectively;

$\dot{\tilde{Y}}$ - experton “return - return”;

$\dot{\tilde{R}}$ - experton “risk - risk”;

$\dot{\tilde{E}}_{Y\to R}$ - experton “return – risk”.

Delayed effects of returns on risks of the shares in the portfolio are evaluated at the fourth stage. This stage includes de-accumulation (to the number “zero”) of the fuzzy matrices of influence of returns on risks, calculation of the mathematical expectation for fuzzy matrices of de-accumulated influences of returns on risks and evaluation of delayed effects of returns on risks. The first activity refers to the experton of influences of returns on risks and to the fuzzy matrix of combined influences of I and II generations of returns on risks. The second activity is aimed at taking into account possibilities of occurring of de-accumulated evaluations of the return influence on risk. It is applied in respect to confidence fours of the experton of de-accumulated influences and of the fuzzy matrix of de-accumulated combined influences of I and II generations as well as in respect to confidence fours of the portfolio in these experton and fuzzy matrix. Confidence fours of the mathematical expectations are substituted by their representative numbers, which are systematized in so-called "representative matrices".

Delayed effects are defined by:

1. formation of the difference between elements of the representative matrices of mathematical expectations for returns influence on risks (see formula (9) and for combined influences of I and II generations; and
2. subsequent definition as delayed effects of the differences, which are equal to or higher than given constant (*c*), belonging to the interval *(0,1*] (see formula (10).

$$(9)\quad D\varepsilon_H = \left|D\varepsilon^{A_i}_{Aj,H}\right| = \varepsilon^{(2),A_i}_{A_j,H} - \varepsilon^{(1),A_i}_{Aj,H}\ \ D\varepsilon^{A_i}_{Aj,H}, \varepsilon^{(2),A_i}_{A_j,H}, \varepsilon^{(1),A_i}_{Aj,H} \in [0,1]$$

$$(10)\ Dde^{A_i}_{A_j} = D\varepsilon^{A_i}_{Aj,H}\ \ for\ D\varepsilon^{A_i}_{Aj,H} \geq c, c \in (0,1]$$

where:

$D\varepsilon_H$ - is the matrix of the difference of mathematical expectations for returns influence on risks,

$\varepsilon^{(1),A_i}_{A_j,H}$ - representative number of the mathematical expectation of de-accumulated return influence of the share A_i on the risk of the share Aj

$\varepsilon^{(2),A_i}_{A_j,H}$ - representative number of the mathematical expectation for de-accumulated combined influence of I and II generations of the return influence of share A_i on the risk of share Aj;

$Dde^{A_i}_{A_j}$ - delayed effect of the return influence of the share A_i on the risk of the share Aj.

4. RESULTS

This part of the article covers only an illustration of the proposed fuzzy model for evaluation of investment portfolios. The approbation of the model with real data suggests a separate survey. According to the authors its results can hardly be expressed in this publication because of its limited volume.

The approbation of the suggested approach was accomplished for three portfolios, each consisting of four shares (A1 to A4). Shares in all three portfolios are of the same kind, but participate in portfolios with different weightings.

The results for the return influence on the risk of the shares in portfolios 1, 2 and 3 are shown in Tables 1, 2 and 3 respectively. Graphical presentations of the results for the return influence of the three portfolios on the risk of the share A1 are done in figure 1 (see tables 1, 2 and 3, column "Share A1", row "Portfolio…").

Tab. 1 – Mathematical Expectations for Portfolio 1. Source: own creation

Shares	Mathematical expectation for the evaluations of return influence on risk of the shares of portfolio 1															
	Share A1				Share A2				Share A3				Share A4			
	min for a=0	min for a=1	max for a=1	max for a=0	min for a=0	min for a=1	max for a=1	max for a=0	min for a=0	min for a=1	max for a=1	max for a=0	min for a=0	min for a=1	max for a=1	max for a=0
Share A1																
evaluations	0,465	0,534	0,900	1,000	0,399	0,467	0,900	0,967	0,465	0,534	0,833	1,000	0,399	0,467	0,800	0,967
representative number	0,72				0,68				0,70				0,65			
Share A2																
evaluations	0,367	0,534	0,833	0,900	0,367	0,499	0,800	0,899	0,367	0,534	0,833	0,900	0,367	0,499	0,800	0,934
representative number	0,67				0,64				0,67				0,65			
Share A3																
evaluations	0,567	0,600	0,833	0,967	0,467	0,500	0,767	0,899	0,466	0,600	0,833	0,967	0,567	0,600	0,800	0,933
representative number	0,73				0,65				0,72				0,72			
Share A4																
evaluations	0,400	0,533	0,833	0,934	0,400	0,533	0,833	0,899	0,400	0,533	0,767	0,934	0,400	0,533	0,800	0,900
representative number	0,68				0,67				0,66				0,66			
Portfolio 1 (shares A1 till A4)																
evaluations	0,450	0,550	0,850	0,950	0,408	0,500	0,825	0,916	0,425	0,550	0,817	0,950	0,433	0,525	0,800	0,934
representative number	0,70				0,66				0,68				0,67			

Tab. 2 – Mathematical Expectations for Portfolio 2. Source: own creation

Shares	Mathematical expectation for the evaluations of return influence on risk of the shares of portfolio 2															
	Share A1				Share A2				Share A3				Share A4			
	min for a=0	min for a=1	max for a=1	max for a=0	min for a=0	min for a=1	max for a=1	max for a=0	min for a=0	min for a=1	max for a=1	max for a=0	min for a=0	min for a=1	max for a=1	max for a=0
Share A1																
evaluations	0,367	0,500	0,833	0,900	0,567	0,600	0,767	0,899	0,567	0,600	0,733	0,733	0,466	0,600	0,833	0,899
representative number	0,66				0,70				0,66				0,71			
Share A2																
evaluations	0,499	0,567	0,800	0,967	0,533	0,600	0,800	0,900	0,533	0,600	0,767	0,866	0,499	0,600	0,800	0,900
representative number	0,70				0,71				0,69				0,70			
Share A3																
evaluations	0,433	0,533	0,800	0,933	0,501	0,567	0,834	1,000	0,501	0,567	0,834	1,000	0,501	0,567	0,800	0,899
representative number	0,67				0,72				0,72				0,69			
Share A4																
evaluations	0,499	0,567	0,799	0,967	0,567	0,600	0,767	0,866	0,567	0,600	0,700	0,767	0,500	0,600	0,799	0,867
representative number	0,70				0,69				0,66				0,69			
Portfolio 2 (shares A1 till A4)																
evaluations	0,450	0,542	0,808	0,942	0,542	0,592	0,792	0,916	0,542	0,592	0,759	0,842	0,492	0,592	0,808	0,891
representative number	0,68				0,70				0,68				0,70			

Tab. 3 – Mathematical Expectations for Portfolio 3. Source: own creation

Shares	Mathematical expectation for the evaluations of return influence on risk of the shares of portfolio 3															
	Share A1				Share A2				Share A3				Share A4			
	min for a=0	min for a=1	max for a=1	max for a=0	min for a=0	min for a=1	max for a=1	max for a=0	min for a=0	min for a=1	max for a=1	max for a=0	min for a=0	min for a=1	max for a=1	max for a=0
Share A1																
evaluations	0,567	0,600	0,867	0,967	0,600	0,600	0,799	0,833	0,600	0,600	0,833	0,967	0,500	0,600	0,867	0,934
representative number	0,74				0,71				0,74				0,73			
Share A2																
evaluations	0,567	0,700	0,867	0,967	0,533	0,700	0,799	0,867	0,567	0,700	0,767	0,867	0,567	0,600	0,867	0,967
representative number	0,78				0,73				0,73				0,74			
Share A3																
evaluations	0,567	0,600	0,833	0,900	0,533	0,600	0,834	0,899	0,567	0,600	0,833	0,900	0,567	0,600	0,767	0,833
representative number	0,72				0,72				0,72				0,69			
Share A4																
evaluations	0,567	0,700	0,833	0,967	0,533	0,700	0,767	0,867	0,567	0,700	0,767	0,867	0,567	0,600	0,833	0,967
representative number	0,77				0,72				0,73				0,73			
Portfolio 3 (shares A1 till A4)																
evaluations	0,567	0,650	0,850	0,950	0,550	0,650	0,800	0,867	0,575	0,650	0,800	0,900	0,550	0,600	0,834	0,925
representative number	0,75				0,72				0,73				0,72			

Tab. 4 – Mathematical Expectations for Delayed Effects of Portfolio 3. Source: own creation

Shares	Mathematical expectation for the evaluations of delayed effects of return on risk of the shares of portfolio 3			
	Share A1	Share A2	Share A3	Share A4
A1	0,045	0,055	0,079	0,148
A2	**0,208**	0,173	0,148	0,115
A3	0,102	0,107	0,072	0,129
A4	0,137	0,052	0,048	0,063
Portfolio 3	0,123	0,089	0,079	0,114

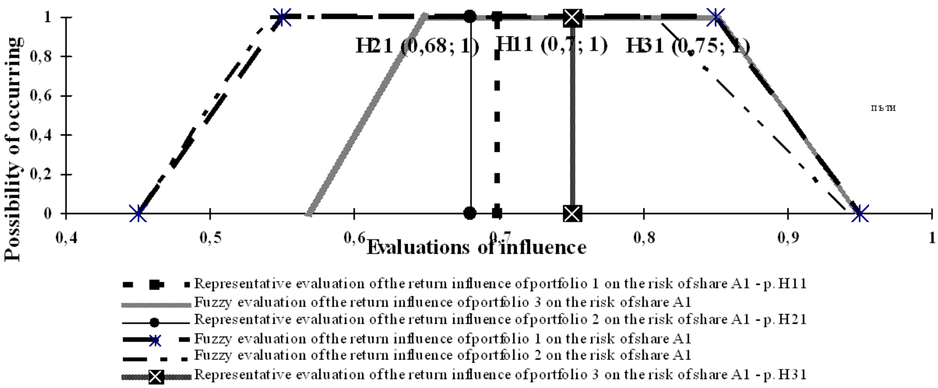

Fig. 1 – Fuzzy evaluations of return influence of portfolio 1, 2 and 3 on the risk of share A1. Source: Own creation.

5. DISCUSSION

It is obvious from tables 1 to 3 that the three portfolios are characterized by high degree of the return influence on the risk belonging to the range [*0,66;0,75*]. The highest result is that of portfolio 3 (table 3). Therefore other things being equal the choice is definitely for portfolio 3.

The results of the approach approbation show that the delayed effects of returns on risks in the evaluation of combined influences of I and II generation for the three portfolios are lower than 0,21. These delayed effects are defined as very low or negligible. Table 4 presents evaluations of delayed effects of portfolio 3. That is the portfolio with the highest evaluations of delayed effects among the three portfolios (see table 4, row "Share A2" and column "Share A1"). This result is logical given that portfolio 3 is the portfolio with the highest degree of return influence on the risk of the shares.

6. CONCLUSION

This paper presents a new approach for evaluating investment portfolios through fuzzy tools of the theory of confidence intervals and theory of fuzzy subsets. The approach consists in determining mutual and hidden influences between the significant variables of the investment portfolio in which evaluations of the influences are described by fuzzy trapezoidal numbers and are aggregated by mathematical operations on fuzzy incidence matrices and fuzzy functions "experton".

General concept of the investment portfolio is reviewed in the paper. Phases of the process of managing the investment portfolio are determined. Important remarks about realization of a proposed optimal solution to a portfolio problem are pointed out. Need for fuzzy approaches to solve this task in the context of complexity and abnormality of the financial markets is substantiated. Concept of the fuzzy approach suggested by the authors of the article is presented. Tools and stages of the methods for the implementation of the approach are characterized. Results of the approach approbation are systematized and analyzed. The approbation is realized through the case data and is aimed only to demonstrate the approach and its applicability.

According to the authors the approach suggested could also be used as a base for comparison and/or ranking different portfolios. Last but not least the used experts' evaluations may be aggregated results from other approaches for portfolio management. Thus, the approach could be described as a universal tool to combine several methods.

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